Modeling And Pricing Event Risk
Part II - The Discount Rate

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In this white paper we will build a model to calculate the discount rate applied to expected cash flow to be received at some future time t given the probability that a one-time event that materially reduces cash flow may occur in the future. To assist us in this endeavor we will use the following hypothetical problem...

Our Hypothetical Problem

We are tasked with calculating the discount rate to be applied to expected cash flow to be received over the time interval $[0, \infty]$. We will use the following model assumptions...

Table 1: Model Assumptions

Symbol	Description	Value
λ	Hazard rate (From Part I)	0.1250
J	Log of one minus jump size (From Part I)	-0.9163
r_f	Risk-free interest rate (%)	3.0000
r_m	Average annual market return (%)	10.0000
σ_m	Average annual market return volatility (%)	15.0000
σ_c	Average annual proxy company return volatility (%)	25.0000
$\rho_{m,c}$	Correlation of market and proxy company returns	0.7000

We will use our model to answer the following questions:

Question 1: What is the annualized cash flow growth rate variance at year zero?
Question 2: What is the annualized cash flow growth rate variance at year five?
Question 3: What is the discount rate applied to expected cash flow received at the end of year five?

The Market Price Of Risk

We will define the variable κ_c to be the discount rate applied to the proxy company's expected cash flows over the time interval $[0, \infty]$ assuming that the probability of a jump event is zero. Using the parameters from Table 1 above the equation for the discount rate assuming no jump is the following CAPM equation...

$$\kappa_c = r_f + \beta_c \, (r_m - f_f) \tag{1}$$

Using the parameters from Table 1 above the regression coefficient β in Equation (1) above is defined as...

$$\beta_c = \frac{\sigma_c}{\sigma_m} \, \rho_{m,c} \tag{2}$$

We will define the variable π_c to be the market price of risk, which is defined as the required rate of return per unit of proxy company return volatility. Using Equations (1) and (2) above we will define the market price of risk to be the following equation...

$$\pi_c = \frac{\rho_{m,c}}{\sigma_m} \, (r_m - f_f) \tag{3}$$

Using Equations (2) and (3) above we can rewrite Equation (1) above as...

$$\kappa_c = r_f + \pi_c \, \sigma_c \tag{4}$$

Time-Dependent Return Variance

In Part I of this series we defined the random variable R_t to be log return, which is defined as the change in the log of random annualized cash flow over the time interval $[0, t]$. We will define the function $f(t)$ to be the variance of the distribution of R_t. The equation for log return variance is... [1]

$$f(t) = \sigma^2 t + J^2 \bigg[\mathrm{Exp}\bigg\{ -\lambda\, t\bigg\} - \mathrm{Exp}\bigg\{ -2\,\lambda\, t\bigg\}\bigg] \tag{5}$$

Using Appendix Equation (17) below the equation for the derivative of Equation (5) above with respect to time is...

$$\frac{\delta f(t)}{\delta t} = \sigma^2 + J^2 \bigg[-\lambda\,\mathrm{Exp}\bigg\{ -\lambda\, t\bigg\} + 2\,\lambda\,\mathrm{Exp}\bigg\{ -2\,\lambda\, t\bigg\}\bigg] \tag{6}$$

We will define the function ϕ_t^2 to be time-dependent log return variance, which is defined as the log return variance $f(t)$ divided by time. Using Equation (5) above the equation for the time-dependent log return variance is...

$$\phi_t^2 = \frac{f(t)}{t} = \sigma^2 + J^2 \bigg[\mathrm{Exp}\bigg\{ -\lambda\, t\bigg\} - \mathrm{Exp}\bigg\{ -2\,\lambda\, t\bigg\}\bigg]\, t^{-1} \tag{7}$$

We will need the time-dependent log return variance at time zero. Note that at time zero the function ϕ_t^2 as defined by Equation (7) above is undefined. This statement in equation form is...

$$\phi_0^2 = \frac{f(0)}{0} = \frac{0}{0} = \text{Undefined} \tag{8}$$

Using L'Hospital's Rule we can rewrite Equation (8) above as...

$$\phi_0^2 = \lim_{t \to 0} \phi_t^2 = \frac{\delta f(t)}{\delta t} \bigg/ \frac{\delta t}{\delta t} = \frac{\delta f(t)}{\delta t} \tag{9}$$

Using Equation (6) above the solution to Equation (9) above is...

$$\phi_0^2 = \sigma^2 + J^2 \bigg[-\lambda\,\mathrm{Exp}\bigg\{ -\lambda \times 0\bigg\} + 2\,\lambda\,\mathrm{Exp}\bigg\{ -2\,\lambda \times 0\bigg\}\bigg] = \sigma^2 + J^2 \bigg[-\lambda + 2\,\lambda\bigg] = \sigma^2 + J^2 \lambda \tag{10}$$

Time-Dependent Discount Rate

We will define the variable κ_t to be the time-dependent discount rate applied to the enterprise's expected cash flows received at time t. Using Equations (4) and (7) above the equation for the time-dependent discount rate is...

$$\kappa_t = r_f + \pi\, \phi_t \tag{11}$$

Using Equation (3) above and the parameters from Table 1 above the parameters to Equation (11) above are...

$$r_f = 0.03 \text{ ...and... } \pi = \frac{0.70}{0.15} \times (0.10 - 0.03) = 0.3267 \tag{12}$$

Using the parameter definitions in Equation (13) above we can rewrite Equation (11) above as...

$$\kappa_t = 0.03 + 0.3267 \times \phi_t \tag{13}$$

The Answers To Our Hypothetical Problem

Question 1: What is the annualized cash flow growth rate variance at year zero?

Using Equation (10) above and the parameters from Table 1 above the annualized cash flow growth rate variance at year zero is...

$$\phi_0^2 = 0.25^2 + (-0.9163)^2 \times 0.1250 = 0.1675 \tag{14}$$

Question 2: What is the annualized cash flow growth rate variance at year five?

Using Equation (7) above and the parameters from Table 1 above the annualized cash flow growth rate variance at year zero is...

$$\phi_5^2 = 0.25^2 + (-0.9163)^2 \times \left[\mathrm{Exp}\left\{-0.1250 \times 5\right\} - \mathrm{Exp}\left\{-2 \times 0.1250 \times 5\right\}\right] \times \frac{1}{5} = 0.1043 \tag{15}$$

Question 3: What is the discount rate applied to expected cash flow received at the end of year five?

Using Equations (13) above and the answer to Question 2 above the discount rate applied to expected cash flow received at the end of year five is...

$$\kappa_5 = 0.03 + 0.3267 \times \sqrt{0.1043} = 0.1355 \tag{16}$$

References

[1] Gary Schurman, *Modeling And Pricing Event Risk - Part I,* August, 2017.

Appendix

A. The solution to the following equation is...

$$\frac{\delta}{\delta t}\left[\mathrm{Exp}\left\{-\lambda\,t\right\} - \mathrm{Exp}\left\{-2\,\lambda\,t\right\}\right] = -\lambda\,\mathrm{Exp}\left\{-\lambda\,t\right\} + 2\,\lambda\,\mathrm{Exp}\left\{-2\,\lambda\,t\right\} \tag{17}$$